arXiv:hep-ph/0106101v1 9 Jun 2001

Neutrino Oscillations in Moving and Polarized Matter under the Influence of Electromagnetic Fields

A.E. Lobanov, A.I. Studenikin[1]
Department of Theoretical Physics, Moscow State University, 119899 Moscow, Russia

Abstract

Within the recently proposed [1–3] Lorentz invariant formalism for description of neutrino spin evolution in presence of an arbitrary electromagnetic fields matter motion and polarization effects are considered. It is shown that in the case of matter moving with relativistic speed parallel to neutrino propagation, matter effects in neutrino spin (and also flavour) oscillations are suppressed. In the case of relativistic motion of matter in the opposite direction in respect to neutrino propagation, sufficient increase of effects of matter in neutrino oscillations is predicted. These phenomena could have important consequences in different astrophysical environments.

In [1–3] the Lorentz invariant formalism for neutrino motion in non-moving and isotropic matter under the influence of an arbitrary configuration of electromagnetic fields have been developed. We have derived the neutrino spin evolution Hamiltonian that accounts not only for the transversal to the neutrino momentum components of electromagnetic field but also for the longitudinal components. With the using of the proposed Hamiltonian it is possible to consider neutrino spin precession in an arbitrary configuration of electromagnetic fields including those that contain strong longitudinal components. We have also considered the new types of resonances in the neutrino spin precession $\nu_L \leftrightarrow \nu_R$ that could appear when neutrinos propagate in matter under the influence of different electromagnetic field configurations. Within the proposed approach the parametric resonance of neutrino oscillations in electromagnetic wave field with periodically varying time-dependent amplitude has been also studied [4].

In the studies [1–3] of the neutrino spin evolution we have focused mainly on description of influence of different electromagnetic fields, while modelling the matter we confined ourselves to the most simple case of non-moving and unpolarised matter. Now we should like to go further (see also [5]) and to generalize our approach for the case of moving and polarized homogeneous

[1]E-mail: studenik@srdlan.npi.msu.su

matter. In this paper we develop the covariant description of neutrino oscillations in moving and polarized matter under the influence of electromagnetic fields. This approach is valid for accounting of matter motion and polarization for arbitrary speed of matter. It is shown for the first time that in the case of relativistic motion of matter, the value of effects of matter in neutrino oscillations (spin oscillations as well as flavour oscillations) sufficiently depends on direction of matter motion in respect to neutrino propagation.

It should be noted here that effects of matter polarization in neutrino oscillations were considered previously in several papers (see, for example, [6, 7] and references therein). However, the used in refs. [6, 7] procedure of accounting for the matter polarization effect does not enable one to study the case of matter motion with relativistic speed. Within our approach we can reproduce corresponding results of [6, 7] in the case of matter which is slowly moving or is at rest.

To derive the equation for the neutrino spin evolution in electromagnetic field $F_{\mu\nu}$ in moving and polarized matter we again start from the Bargmann-Michel-Telegdi (BMT) equation [8] for the spin vector S^{μ} of a neutral particle that has the following form

$$\frac{dS^{\mu}}{d\tau}=2\mu\{F^{\mu\nu}S_{\nu}-u^{\mu}(u_{\nu}F^{\nu\lambda}S_{\lambda})\}+2\epsilon\{\tilde{F}^{\mu\nu}S_{\nu}-u^{\mu}(u_{\nu}\tilde{F}^{\nu\lambda}S_{\lambda})\}, \qquad (1)$$

This form of the BMT equation corresponds to the case of the particle moving with constant speed, $\vec{\beta}=const$, $(u_{\mu}=(\gamma,\gamma\vec{\beta}), \gamma=(1-\beta^{2})^{-1/2})$, in presence of an electromagnetic field $F_{\mu\nu}$. Here μ is the fermion magnetic moment and $\tilde{F}_{\mu\nu}$ is the dual electromagnetic field tensor. The neutrino spin vector satisfies the usual conditions, $S^{2}=-1$ and $S^{\mu}u_{\mu}=0$. Equation (1) covers also the case of a neutral fermion having static non-vanishing electric dipole moment, ϵ. Note that the term proportional to ϵ violates T invariance.

The BMT spin evolution equation (1) is derived in the frame of electrodynamics, the model which is P invariant. Our aim is to generalize this equation for the case when effects of various neutrino interactions (for example, weak interaction for which P invariance is broken) with moving and polarized matter are also taken into account. Effects of possible P nonconservation and nontrivial properties of matter (i.e., its motion and polarization) have to be reflected in the equation that describes the neutrino spin evolution in an electromagnetic field.

The Lorentz invariant generalization of eq.(1) for our case can be obtained

by the substitution of the electromagnetic field tensor $F_{\mu\nu} = (\vec{E}, \vec{B})$ in the following way:

$$F_{\mu\nu} \to F_{\mu\nu} + G_{\mu\nu}. \tag{2}$$

In evaluation of the tensor $G_{\mu\nu}$ we demand that the neutrino evolution equation has to be linear over the neutrino spin vector S_μ, electromagnetic field $F_{\mu\nu}$, and such characteristics of matter (which is composed of different fermions, $f = e,\ n,\ p...$) as fermions currents

$$j_f^\mu = (n_f, n_f \vec{v}_f), \tag{3}$$

and fermions polarizations

$$\lambda_f^\mu = \Big(n_f(\vec{\zeta}_f \vec{v}_f), n_f \vec{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}}\Big). \tag{4}$$

Here n_f, $\vec{v}_f$, and $\vec{\zeta}_f$ $(0 \leq |\vec{\zeta}_f|^2 \leq 1)$ denote, respectively, the number density of the background fermions f, the speed of the reference frame in which the mean momentum of fermions f is zero, and the mean value of the polarization vectors of the background fermions f in the above mentioned reference frame. Note that, as it follows from eq.(4), if a component f of matter is not moving, $\vec{v}_f = 0$, the fermion f polarization is expressed as,

$$\lambda_f^\mu = (0, n_f \vec{\zeta}_f). \tag{5}$$

The mean value of the background fermion f polarization vector, $\vec{\zeta}_f$, is determined by the two-step averaging procedure. The polarization of the moving fermion is described by the relativistic generalization of the spin operator:

$$\vec{O} = \gamma_0 \vec{\Sigma} - \gamma_5 \frac{\vec{p}}{p_0} - \gamma_0 \frac{\vec{p}(\vec{p}\vec{\Sigma})}{p_0(p_0 + m)}, \tag{6}$$

where

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix},$$

$\vec{\sigma}$ is the Pauli matrices and $\vec{p}, p_0, m$ are, respectively, the fermion momentum, energy and mass. At the first step the average of the fermion spin

operator $\vec{O}$ have to be evaluated over the fermion quantum state in a given electromagnetic field,

$$< \vec{O} >= \int \Psi_f^+(x)\vec{O}\Psi_f(x)dx, \tag{7}$$

here $\Psi_f(x)$ is the exact solution of the Dirac equation for the fermion accounting for the external electromagnetic field.

The second-step averaging is performed over the fermion statistical distribution density function, $\rho_f(\{n\})$:

$$\vec{\zeta}_f = \frac{\sum\limits_{\{n\}} < \vec{O} > \rho_f(\{n\})}{\sum\limits_{\{n\}} \rho_f(\{n\})}. \tag{8}$$

In the case when the background fermions f can be described as an ideal gas, $\rho_f(\{n\})$ is nothing but the Fermi-Dirac distribution function.

For each type of the fermions f there are only three vectors, u^f_μ, j^f_μ, and λ^f_μ, using which the tensor $G_{\mu\nu}$ have to be constructed. If j^f_μ and λ^f_μ are slowly varying functions in space and time (this condition is similar to one imposed on the electromagnetic field tensor $F_{\mu\nu}$ in the derivation of the BMT equation) then one can construct only four tensors (for each of the fermions f) linear in respect to the characteristics of matter:

$$G_1^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda}u_\lambda j_\rho, \quad G_2^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda}u_\lambda \lambda_\rho, \tag{9}$$

$$G_3^{\mu\nu} = u^\mu j^\nu - j^\mu u^\nu, \quad G_4^{\mu\nu} = u^\mu \lambda^\nu - \lambda^\mu u^\nu. \tag{10}$$

Thus, in general case of neutrino interaction with different background fermions f we introduce for description of matter effects antisymmetric tensor

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda}g^{(1)}_\rho u_\lambda - (g^{(2)\mu}u^\nu - u^\mu g^{(2)\nu}), \tag{11}$$

where

$$g^{(1)\mu} = \sum_f \rho^{(1)}_f j^\mu_f + \rho^{(2)}_f \lambda^\mu_f, \quad g^{(2)\mu} = \sum_f \xi^{(1)}_f j^\mu_f + \xi^{(2)}_f \lambda^\mu_f. \tag{12}$$

Summation is performed over the fermions f of the background. The explicit expressions for the coefficients $\rho^{(1),(2)}_f$ and $\xi^{(1),(2)}_f$ could be found if the particular model of neutrino interaction is chosen. In the usual notations the antisymmetric tensor $G_{\mu\nu}$ can be written in the form,

$$G_{\mu\nu} = (- \vec{P}, \ \vec{M}), \tag{13}$$

where

$$\vec{M} = \gamma\{(g_0^{(1)}\vec{\beta} - \vec{g}^{(1)}) - [\vec{\beta} \times \vec{g}^{(2)}]\}, \ \vec{P} = -\gamma\{(g_0^{(2)}\vec{\beta} - \vec{g}^{(2)}) + [\vec{\beta} \times \vec{g}^{(1)}]\}. \quad (14)$$

It worth to note that the substitution (2) implies that the magnetic $\vec{B}$ and electric $\vec{E}$ fields are shifted by the vectors $\vec{M}$ and $\vec{P}$, respectively:

$$\vec{B} \to \vec{B} + \vec{M}, \quad \vec{E} \to \vec{E} - \vec{P}. \quad (15)$$

In the case of non-moving, $\vec{v}_f = 0$, and unpolarized, $\vec{\zeta}_f = 0$, matter we get, in agreement with our previous result [1–3],

$$G_{\mu\nu} = \Big(\gamma\vec{\beta}\sum_f \xi_f^{(1)} n_f, \gamma\vec{\beta}\sum_f \rho_f^{(1)} n_f\Big). \quad (16)$$

We finally arrive to the following equation for the evolution of the three-dimensional neutrino spin vector $\vec{S}$ accounting for the direct neutrino interaction with electromagnetic field $F_{\mu\nu}$ and matter (which is described by the tensor $G_{\mu\nu}$):

$$\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma}\Big[\vec{S} \times (\vec{B}_0 + \vec{M}_0)\Big] + \frac{2\epsilon}{\gamma}\Big[\vec{S} \times (\vec{E}_0 - \vec{P}_0)\Big]. \quad (17)$$

The derivative in the left-hand side of eq.(17) is taken with respect to time t in the laboratory frame, whereas the values $\vec{B}_0$ and $\vec{E}_0$ are the magnetic and electric fields in the neutrino rest frame given in terms of the transversal in respect to the neutrino motion ($\vec{F}_\perp$) and longitudinal ($\vec{F}_\parallel$) fields $\vec{F} = \vec{B}, \vec{E}$ in the laboratory frame,

$$\begin{array}{c} \vec{B}_0 = \gamma(\vec{B}_\perp + \frac{1}{\gamma}\vec{B}_\parallel + \sqrt{1 - \frac{1}{\gamma^2}}[\vec{E}_\perp \times \vec{n}]), \\ \vec{E}_0 = \gamma(\vec{E}_\perp + \frac{1}{\gamma}\vec{E}_\parallel - \sqrt{1 - \frac{1}{\gamma^2}}[\vec{B}_\perp \times \vec{n}]), \vec{n} = \vec{\beta}/\beta. \end{array} \quad (18)$$

The influence of matter on the neutrino spin evolution in eq.(17) is given by the vectors $\vec{M}_0$ and $\vec{P}_0$ which in the rest frame of neutrino can be expressed in terms of quantities determined in the laboratory frame

$$\vec{M}_0 = \gamma\vec{\beta}\Big(g_0^{(1)} - \frac{\vec{\beta}\vec{g}^{(1)}}{1 + \gamma^{-1}}\Big) - \vec{g}^{(1)}, \quad (19)$$

$$\vec{P}_0 = -\gamma\vec{\beta}\Big(g_0^{(2)} - \frac{\vec{\beta}\vec{g}^{(2)}}{1+\gamma^{-1}}\Big) + \vec{g}^{(2)}. \tag{20}$$

Let us determine the coefficients $\rho_f^{(i)}$ and $\xi_f^{(i)}$ in eq.(12) for the particular case of the electron neutrino propagation in moving and polarized electron gas. We consider the standard model of interaction supplied with $SU(2)$-singlet right-handed neutrino ν_R. The neutrino effective interaction Lagrangian reads

$$L_{eff} = -f^{\mu}\Big(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^5}{2}\nu\Big), \tag{21}$$

where

$$f^{\mu} = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^2\theta_W)j_e^{\mu} - \lambda_e^{\mu}\Big). \tag{22}$$

In this case neutrino electric dipole moment vanishes, $\epsilon = 0$, so that the coefficients $\xi_e^{(i)} = 0$, and from the obvious relation, $f_{\mu} = 2\mu g_{\mu}^{(1)}$, it follows

$$\rho_e^{(1)} = \frac{G_F}{2\mu\sqrt{2}}(1+4\sin^2\theta_W), \ \rho_e^{(2)} = -\frac{G_F}{2\mu\sqrt{2}}. \tag{23}$$

If for the neutrino magnetic moment we take the vacuum one-loop contribution [9, 10]

$$\mu_{\nu} = \frac{3}{8\sqrt{2}\pi^2}eG_F m_{\nu},$$

then

$$\rho^{(1)} = \frac{4\pi^2}{3em_{\nu}}(1+4\sin^2\theta_W), \quad \rho^{(2)} = -\frac{4\pi^2}{3em_{\nu}}.$$

We should like to note that solutions of the derived eq.(17) for the neutrino spin evolution in moving and polarized matter and, correspondingly, the neutrino oscillation probabilities and effective mixing angles θ_{eff} can be obtained for different configurations of electromagnetic fields in a way similar to that described in [1–3].

Consider the case of neutrino propagating in the relativistic flux of electrons. Using expressions for the vector $\vec{M}_0$, eqs. (12), (19), we find,

$$\begin{aligned}\vec{M}_0 = n_e\gamma\vec{\beta}\Big\{&(\rho^{(1)} + \rho^{(2)}\vec{\zeta}_e\vec{v}_e)(1-\vec{\beta}\vec{v}_e) + \\ +\rho^{(2)}\sqrt{1-v_e^2}&\Big[\tfrac{(\vec{\zeta}_e\vec{v}_e)(\vec{\beta}\vec{v}_e)}{1+\sqrt{1-v_e^2}} - \vec{\zeta}_e\vec{\beta}\Big] + O(\gamma^{-1})\Big\}.\end{aligned} \tag{24}$$

In the case of slowly moving matter, $v_e \ll 1$, we get

$$\vec{M}_0 = n_e \gamma \vec{\beta}\Big(\rho^{(1)} - \rho^{(2)} \vec{\zeta}_e \vec{\beta}\Big). \tag{25}$$

For the unpolarized matter eq.(25) reproduces the Wolfenstein matter term [11] and confirms our previous result [1–3]. In the opposite case of relativistic flux, $v_e \sim 1$, we find,

$$\vec{M}_0 = n_e \gamma \vec{\beta}\Big(\rho^{(1)} + \rho^{(2)} \vec{\zeta}_e \vec{v}_e\Big)\Big(1 - \vec{\beta}\vec{v}_e\Big). \tag{26}$$

If we introduce the invariant electron number density,

$$n_0 = n_e \sqrt{1 - v_e^2}, \tag{27}$$

then it follows,

$$\vec{M}_0 = n_o \gamma \vec{\beta} \tfrac{(1-\vec{\beta}\vec{v}_e)}{\sqrt{1-v_e^2}}\Big(\rho^{(1)} + \rho^{(2)} \vec{\zeta}_e \vec{v}_e\Big). \tag{28}$$

Thus, in the case of the parallel motion of neutrinos and electrons of the flux, the matter effect contribution to the neutrino spin evolution equation (17) is suppressed. In the case of neutrino and matter relativistic motion (β and $v_e \sim 1$) in opposite directions, the matter term $\vec{M}_0$ gets its maximum value

$$\vec{M}_0^{max} = 2 n_e \gamma \vec{\beta}\Big(\rho^{(1)} + \rho^{(2)} \vec{\zeta}_e \vec{v}_e\Big), \tag{29}$$

which is equal to the matter term derived for the case of slowly moving ($v_e \ll 1$) matter times a factor $\frac{2}{\sqrt{1-v_e^2}}$. Therefor we predict sufficient increase of matter effects in neutrino oscillations for neutrino propagating against the relativistic flux of matter.

Finally, let us consider neutrino spin oscillations in an arbitrary constant magnetic field, $\vec{B} = \vec{B}_\parallel + \vec{B}_\perp$, and moving matter. In the adiabatic approximation for the particular case of electron neutrinos ν_e propagating in matter composed of electrons, $f = e$, the probability of conversion $\nu_L \to \nu_R$ can be written in the form,

$$P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}, \tag{30}$$

$$sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, L_{eff} = \frac{2\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}, \tag{31}$$

where $E_{eff} = 2\mu B_\perp$ (terms $\sim O(\gamma^{-1})$ are ommited here), and

$$\Delta_{eff} = V(1 - \vec{\beta}\vec{v}_e) + \frac{2\mu B_\parallel}{\gamma}, V = \frac{G_F}{\sqrt{2}} n_e. \tag{32}$$

As it is mentioned above, the matter effect in Δ_{eff} can be "eaten" by the relativistic motion of matter. It follows that the condition for maximal mixing of ν_L and ν_R in a magnetic field $\vec{B}$ (which is realised [2] in the vacuum when neutrino is propagating nearly perpendicular to the magnetic field, $B \approx B_\perp, B_\parallel \approx 0$) could be realised also in presence of matter moving with relativistic speed.

From the above discussion on matter effects in neutrino spin oscillations it follows that the similar suppression of matter term exist in neutrino oscillations without change of helicity for the case of matter moving with relativistic speed.

It worth to be noted that if matter is not flavour symmetric (like an "ordinary" matter which contains electrons, neutrons and protons, but not muons and taons) the matter contribution to the effective neutrino potential depends on the neutrino flavour. That is why modification of matter effects in neutrino oscillations will be different not only for different speeds of matter components, but also will be varied for neutrino oscillations between various neutrino flavour states.

In conclusion, we argue that the discussed phenomena of sufficient change of matter effects in neutrino oscillations in case of matter motion with relativistic speed could have important consequences in astrophysics.

Acknowledgements

We thank A.Akhmedov, V.Berezinsky, C.Giunti, V.Naumov, S.Petcov, and A.Smirnov for helpful discussions. We should also like to thank A.Borisov, P.Eminov and P.Nikolaev for consultations on the relativistic quantum statistics.

References

[1] A.Egorov, A.Lobanov, A.Studenikin, in: New Worlds in Astroparticle Physics, ed. by A.Mourao, M.Pimento, P.Sa, World Scientific, Singapore, p.153, 1999; hep-ph/9902447.

[2] A.Egorov, A.Lobanov, A.Studenikin, Phys.Lett.B491 (2000) 137, hep-ph/9910476.

[3] A.Egorov, A.Lobanov, A.Studenikin, in: Results and Perspectives in Particle Physics, ed. by M.Greco, Frascati Physics Series, v. 17, p. 117, 2000.

[4] M.Dvornikov, A.Studenikin, in:New Worlds in Astroparticle Physics, ed. by P.Sa, World Scientific, Singapore, 2001; Phys.At.Nucl.64 (2001) in press; hep-ph/0102099.

[5] M.Dvornikov, A.Egorov, A.Lobanov, A.Studenikin, in: A.Studenikin (Ed.), Particle Physics at the Start of the New Millennium, World Scientific, Singapore, 2001, p. 178; hep-ph/0103015.

[6] H.Nunokawa, V.Semikoz, A.Smirnov, J.W.F.Valle, Nucl.Phys.B501 (1997) 17.

[7] S.Bergmann, Y.Grossman, E.Nardi, Phys.Rev.D60 (1999) 093008.

[8] V.Bargmann, L.Michel, V.Telegdi, Phys.Rev.Lett.2 (1959) 435.

[9] B.Lee, R.Shrock, Phys.Rev.16 (1977) 1444.

[10] K.Fujikawa, R.Shrock, Phys.Rev.Lett.45 (1980) 963.

[11] L.Wolfenstein, Phys.Rev.D17 (1978) 2369.